Extending the Maximum Effective Range of Small Arms

By Bryan Litz

In a previous issue of Precision Shooting magazine, there appeared an article describing a procedure for calculating the maximum effective range of small arms. In the present article, I'll attempt to expand on that work in two ways. The first objective is to show how the hit probability diminishes as the Maximum Effective Range (MER) is exceeded. Furthermore, ideas are presented for how to increase in maximum effective range by understanding and applying corrections for so called "6 degree of freedom effects".

Setting the Stage

I tried to write this second article like a "Rocky" movie. It should be able to stand alone, but it's a lot better if you've seen (read) the prequel(s).

For those readers interested in the foundations of the 6 degree of freedom projectile modeling methods and tools used for studying Maximum Effective Range (MER), I will to reference you to my first article which appeared in the last issue of Precision Shooting. However, a short summary of the fundamental ideas presented in that issue will be rehashed now, just like the beginning of every Rocky movie....

Aerodynamic and mass properties prediction codes are used to generate data about a particular bullet. That data is fed into a computer program, which applies the initial conditions and solves the 6-degree of freedom (6-DOF) equations of free-flight motion for the projectile. The 6 degrees of freedom are: Translation in 3 directions (front to back, side to side, up and down, or x,y and z) and rotation (roll, pitch and yaw). The results of running such a program are much more detailed than the standard "drop and drift" ballistics programs offered by classical ballistics software. Using 6-DOF simulation, one can calculate things like: Coriolis acceleration, aerodynamic jump, lateral throw-off, gyroscopic drift, yaw dependant drag, etc. One application of the 6-DOF simulation is the ability to perform detailed analysis of the Maximum Effective Range (MER) of a weapon system.

In order to find the MER of a rifle, one must first decide what conditions will be used to declare a valid, or 'effective' shot. These are the "MER conditions". There are accuracy MER conditions and lethality MER conditions. Basically, if the bullets can be guaranteed to impact a target within the "kill zone" (accuracy requirement), while retaining the required energy (lethality condition), then the target is said to be within the MER of the weapon. If any combination of assumed "field variables" causes a shot to impact outside of the prescribed "kill zone", or if the bullet hits the kill zone but doesn't have enough energy, then the target is said to be outside the MER of the weapon.

Moving on

Time to see who Rocky's fighting this time…

The notion of the MER so far has been described as the maximum range at which the circular kill zone is *guaranteed* to be struck by the bullet. In the previous article, a .243 varmint rifle with "varmint" type MER conditions and field variables was found to have a MER of 248.5 yards. So under the influence of variables expected in the field, the weapon system was guaranteed capable of striking a 6" circle up to a range of 248.5 yards while retaining at least 500 ft-lbs of kinetic energy. So what happens at 250 yards?

What about 275 or 300 yards? It's certainly *possible* to hit the 6" kill zone at these ranges, but how *"probable"* is it?

Hit Probability

Figure 1 is from my previous article. The 4 numbered shots represent the effects of applying extreme combinations of field variables while shooting at a 6-inch target, or a 6-inch "kill zone" that's 300 yards away. The circles around the numbers are there to represent the inherent accuracy of the rifle, in this case, 0.5 MOA. The sort of rectangular area that's enclosed by the 4 corners represents the area that all shots are expected to impact while under the influence of the assumed field variables. Note the size of the impact area and it's relationship to the kill zone. Under the influence of the established field variables, the impact area lays entirely within the 6" kill zone *up to a range of 248.5 yards*. At ranges beyond 248.5 yards, a portion of the impact area will lie outside of the kill zone as shown on the 300-yard target in Figure 1. In Figure 1, you can see that almost all of the impact area is within the kill zone. But the combination of field variables that results in shot number 3 will cause impacts outside of the kill zone. In order to calculate the *hit probability*, one must calculate the percentage of the impact area that lies within the kill zone. In the case of the 300-yard target, 86% of the expected impact area lies within the kill zone. That means that under the influence of the assumed field variables, the weapon system that's described has an 86% chance of hitting the 6-inch kill zone.

86% hit probability

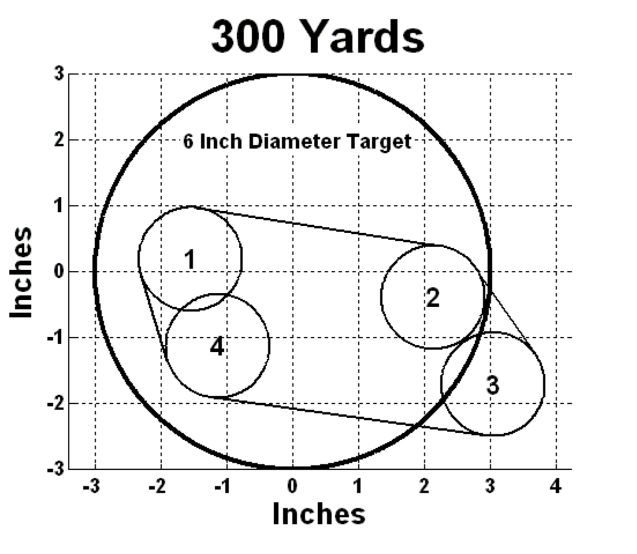


Figure 1. Effect of field variables at 300 yards, corrected for 5 mph wind

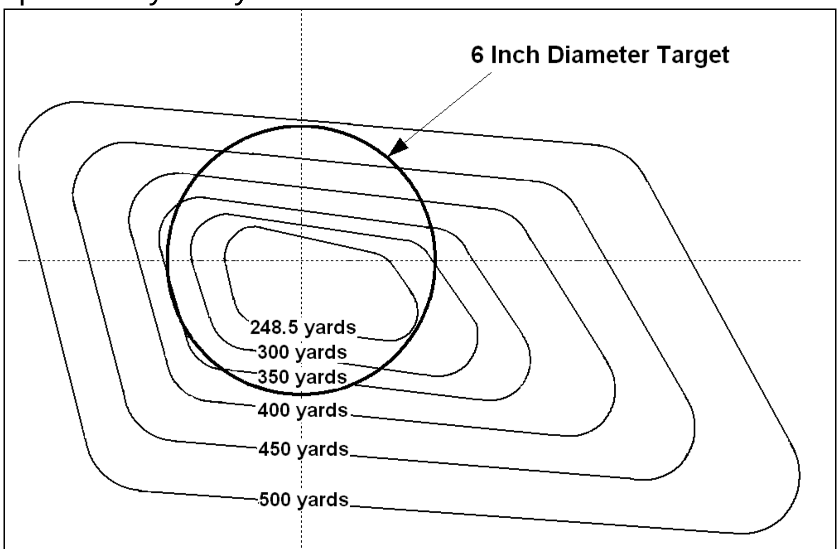
That's the basic explanation; however there are some complicating factors involving the statistics of the assumed field variables that will be addressed later. There are also some terminal ballistics issues regarding the lethality condition that need to be investigated.

Taking a closer look at the *lethality condition*

So the matter of finding hit probability is reduced to a simple geometric exercise of comparing areas. The analysis of areas can be used to find the hit probability, but what about the *kill probability*? After all, that is the point, right? Suppose we calculate that our .243 has a 70% probability of hitting the 6-inch kill zone of an animal at some range. What is the probability that the 80-grain varmint bullet will kill the animal? And this, my friends, is my exit. I have no idea how to figure out the chances of a shot actually killing a target. It's probably safe to say that such a calculation is impossible. If after reading the following advice, you're still not satisfied, then I suggest you take up the "kill criteria" with God.

Having failed in my attempts to actually *calculate death*, I've settled on the following, less metaphysical guidelines for enforcing the lethality condition. Simply identify a value of kinetic energy that you feel is sufficient for the selected bullet and its intended target. At the range where the impact energy has dropped below the selected value, the shot is no longer considered "lethal". I should say that this type of terminal ballistic analysis of bullet performance is one of my weak areas of understanding. There may be simple "rules of thumb" relating bullet performance, kinetic energy, and lethality. If anyone knows how to figure that stuff out, it would be a valuable addition to this analysis.

Let's look at trying to bound the range with the lethality condition. I've estimated that 500 ft-lbs for the 80-grain varmint bullet with its thin jacket will have sufficient lethality for varmint type targets. This corresponds to a speed of 1679 fps, which at standard sea level conditions and 3000 fps muzzle velocity, occurs at 484 yards. Considering the field variables that affect downrange energy, the 500 ft-lb minimum can be reached as close as 458 yards, or as far as 512 yards. If you're in the mountains, and the standard conditions for 5000 feet altitude are applied (air density = 0.002048 sl/ft^3) the bullet would retain 500 ft-lbs of kinetic energy out to 560 yards. Considering the lingering uncertainty of relating kinetic energy to lethality, I think it's wise to view the lethality condition as simply a kinetic energy limit for now. This will allow one to compare systems using different weight and design bullets, as well as provide a rough range limit for the hit probability analysis.



Range	Hit probability	Kinetic Energy (ft-lbs)
Up to 248.5 yards	100%	915
300	86%	809
350	72%	713
400	49%	627
450	33%	548
500	22%	477

Figure 2. Expected impact area in relation to a 6" diameter kill zone. The impact area that's inside the kill zone is related to the "hit probability".

Figure 2 shows the 6-inch kill zone with expected impact areas corresponding to ranges from 248.5-yards out to 500-yards. The hit probability and kinetic energy are shown for each range.

Before we leave the subject of computing hit probability, I'd like to address several issues regarding statistics. I've claimed that the impact area that's contained in the kill zone compared to the total impact area is the hit probability. But that's assuming that every point within the impact area is *equally likely*. That means that any combination of *field variables* is equally likely. One may argue that a combination of extreme field variables is less likely than a combination of variables closer to average and I would agree. The right way to address this concern is to assign a standard deviation to each field variable. So to describe the wind, one would have to say that the average cross wind component is 5 mph, +-2 mph, with a standard deviation of "x" mph. When that is done, one will find that an impact near the edge of the impact area, caused by an extreme field condition is predictably less likely than an impact near the center. One could then contour the impact area showing where shots are more or less expected to land depending on the statistics assumed for the field variables.

The statistical description of field variables and their effect on hit probability took only one paragraph to illustrate. However, to include that detail into our ballistic analysis would easily double or triple the amount of complexity and work required. Although it would be 'cool' to show a contoured impact area, I question how much value is actually added. One can still compare different systems and the effects of different field variables on effectiveness. In other words, the *trends* of the results are unaffected by ignoring the statistics of field variables. And I also question my own ability to choose representative standard deviations for field variables. There's a time honored common sense rule in engineering that says "don't measure with a micrometer if you plan to cut with a chainsaw." My objective is to show the effectiveness of different systems under different sets of field variables against specific targets. A statistical analysis of filed variables would be like measuring with a micrometer.

The other statistical consideration deals with the variation of lethality within the kill zone. Most areas described as "vital zones" of game animals are neither circular nor equally vulnerable and so we're back to the loaded question of lethality and the *death factor,* I'm out.

Is there a point?

Many of you reading my material on computer simulation may see the whole thing as boring theory. So of what use are these detailed analyses? How can they help us shoot better?

Increasing the effective range of a weapon system

Now that 6 degree of freedom effects and hit probability have been described, we can talk about how to increase the effective range of a weapon system. *By understanding and correcting for 6 degree of freedom effects, one is better able to center the impact area over the kill zone.* It's a basic issue of maximizing accuracy. Most shooters are familiar with the definitions of accuracy and precision. High precision places shots very close to each other into a small "group". High accuracy places shots close to the center of an intended target. Accuracy is generally more important than precision when considering weapon effectiveness with regards to hunting. I'd rather go hunting with a rifle that's perfectly zeroed and shoots 1" groups than a rifle capable of 0.1" groups but isn't zeroed.

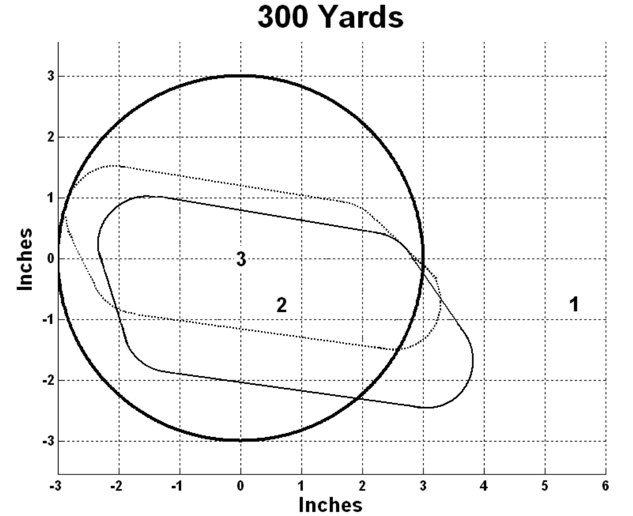
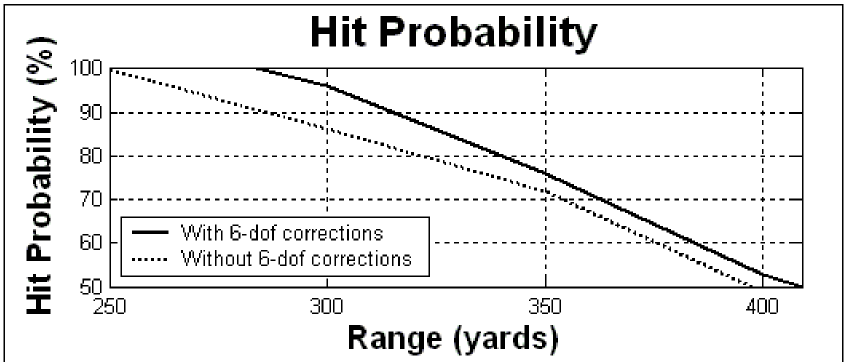


Figure 3. By correcting for 6-dof effects, the impact area is centered over the kill zone

Figure 3 shows the effects of accounting for 6 degree of freedom effects on a 300-yard target. Shot #1 represents average field variables applied to a shot corrected only for gravity drop. Shot #2 represents where the shot lands if one corrects for wind deflection only. Shot #3 shows where you hit if you correct for everything including 6 degree of freedom effects. The solid line outlines the expected impact area without 6 DOF corrections, the dotted line shows where you can expect the impact area to lay if you do correct for 6 DOF effects. Notice that by centering the impact area, you have a better chance that your shots will land within the kill zone. Also, the impact area that's corrected for 6-dof effects is smaller, because the vertical component of the Coreolis acceleration has been corrected for. What are the 6-dof effects that caused shot #2 to hit low right even after the correct gravity drop and wind deflection corrections were applied? Well, it hit *right* because of gyroscopic drift (caused by the bullets tendency to nose right[1] as the bullet drops) and the small horizontal component of Coreolis acceleration. It hit *low* because of aerodynamic jump. Aerodynamic jump is what causes groups to "slant" when shot in varying wind conditions. Basically, when the bullet exits the muzzle into a cross wind, the bullet tries to yaw slightly to align itself with the airflow. When the bullet yaws to the side, gyroscopic action causes it to nose up or down by a small amount depending on the wind direction. This initial yaw has an effect on the trajectory, and is known as aerodynamic jump[2]. The more severe the cross wind, the more pitch the bullet ends up with. Flying to the target at a pitch angle will result in an elevation error that's proportional to crosswind.

[1] A bullet will yaw to the right from a right hand twist barrel, left from a left-twist. The yawing motion is a result of the force applied up on the nose of the bullet as it begins to drop in its trajectory.

[2] When torque is applied to a bullets spin axis, the bullet reacts by rotating its axis 90 degrees from the applied torque, in the direction of rotation. One can observe this gyroscopic action with a top.



Range	Hit probability	
	Without 6 DOF corrections	With 6 DOF corrections
Up to 248.5 yards	100%	100%
284	92%	100%
300	86%	96%
350	72%	76%
400	46%	53%

Figure 4. Illustrating the effect of using 6-dof corrections on hit probability. Maximum guaranteed effective range is increased 14% from 248.5 yards to 284.0 yards.

This is evidenced by the slant of the impact area.

Gyroscopic drift, Coriolis acceleration, and aerodynamic jump are 6-dof effects that classic ballistics software is not able to calculate. Even if the exact corrections are applied for gravity drop[3], and wind deflection, shot #2 still hits 0.9” from center at 300-yards because of 6-dof effects.

Figure 4 shows how much the hit probability is improved by correcting for 6-dof effects.

Applying 6-DOF effects: Is it worth it?

After all that work, we find that for our particular system in its defined application, we are able to increase the range of a guaranteed hit from 248.5 yards to 284 yards for the assumed field variables. That’s a 14% increase in guaranteed hit range. So is it worth all the hassle? That depends.

I’m not writing to sell anything, or to convince you to “drink the 6-dof Kool-aid”. Truth is, I believe that in general, a shooters familiarity with a weapon gained thru personal experience and practice is far more important to the effectiveness of that weapon system than 6-dof ballistic effects. I know that most real hunting situations are usually dictated by “unforeseeable” variables like suddenly moving targets, line of sight obstructions, lack of solid shooting positions, forgetting to take the safety off, or even not having the gun loaded when it’s time to shoot (guilty☹). And so for most of the average hunting and shooting masses, 6-dof effects can be ignored without consequence.

But there are a small percentage of shooters who spend countless hours striving to maximize their chances of connecting with small targets at great distances. Shooters who have learned to avoid all of the common mistakes and pitfalls and mastered all of the basic skills. It’s these elite shooters who stand to benefit from a serious consideration of the details of external ballistics.

Applying 6-DOF effects: How to do it

None of the common ballistics programs are capable of calculating “6 degree of freedom” effects. They all have analytic solvers, which makes them very fast and accurate predictors of gravity drop, drag, and wind drift even for non-standard

[3] I’m contemplating a future article regarding the shortcomings in accuracy of modern ballistics software. Specifically, the consequences of using the G1drag standard to define the ballistic coefficient of long slender bullets will be examined.

atmospheric conditions. However, those analytic solvers cannot calculate “6 degree of freedom effects” such as gyroscopic drift, aerodynamic jump, yaw dependant drag, etc. Also, the G1 drag function used in modern ballistics programs is not an accurate drag profile for long boat-tailed bullets. That’s why the B.C. has to be defined piecewise as a function of velocity. The 6-degree of freedom program uses a numerical solver, which allows the equations of motion to be solved using the actual drag, and not rely on an average fit to a non-representative standard (G1). The problem with the 6-degree of freedom program is speed. It took about 2 minutes for each of the 300-yard trajectories to run on my desktop computer equipped with a 2.08 GHz processor. It’s rather impractical to think that a ballistics program running a full 6-degree of freedom simulation can be run on a palm pilot in the field where it’s needed. However, there is an alternative…

***Run the analytical solution and apply pre-tabulated 6-dof effects to the basic drop and drift results*.** The whole program could run at practically the same speed and provide corrections resulting in more centered shots. The pre-tabulated 6-dof effects would need to be very specialized for a particular shooting system. For example, drift would depend on twist rate and latitude as well as wind speed and direction. Elevation would depend on wind drift and firing direction as well as muzzle velocity, range, gravity, etc.

There are two ways to accomplish the corrections.

1. Create custom tables for each specific shooting system
2. Create rules of thumb based on the class of shooting system.

Option 2 has a much better chance of success due to the difficulty in calculating 6-DOF corrections. For example, we should avoid the challenge of creating gyroscopic drift correction tables for 1000’s of combinations of bullet caliber, muzzle velocity, twist rate, and atmospheric conditions, all of which would be strikingly similar. Instead there would be a simple equation that accounts for all of the relevant variables and gives an answer that’s close for the entire class. You won’t know if the result is ***exactly*** right for your particular bullet/twist, but ***it’s better to apply a correction that’s 80% accurate than to apply no correction at all.***

Class based rules of thumb (equations) could be devised for most of the other important 6-DOF effects and embedded into the existing ballistics programs. The user could choose to apply the corrections or not.

Conclusion

Current trends in precision long range rifle fire are pushing at every corner of the envelope. Many times, great effort and a lot of money is spent just trying to improve the system by a couple percent. Incorporating the predictable 6-dof details of a bullets flight into a modern ballistics program is an inevitable step towards more successful long range shooting.

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